TRINITY COLLEGE

Year 12 Mathematics Applications
Test 3 2022

Section 1 Calculator Free

Networks

STUDENT'S NAME Solutions.

DATE: Wednesday 11th May 2022 **TIME:** 40 minutes **MARKS**: 40

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

A connected planar network has 6 edges and 5 faces.

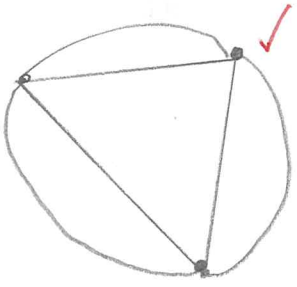
(a) How many vertices must it have? [2]

$v + f - e = 2$

$v + 5 - 6 = 2$ ✓ substitution

$v = 3$ ✓ ans.

(b) Draw a planar graph with 6 edges and 5 faces. [2]



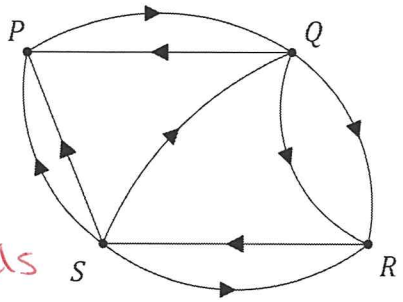
✓ 3 vertices

✓ planar graph.

2. (6 marks)

Calculate

(a) Construct the adjacency matrix for the following planar graph. [2]

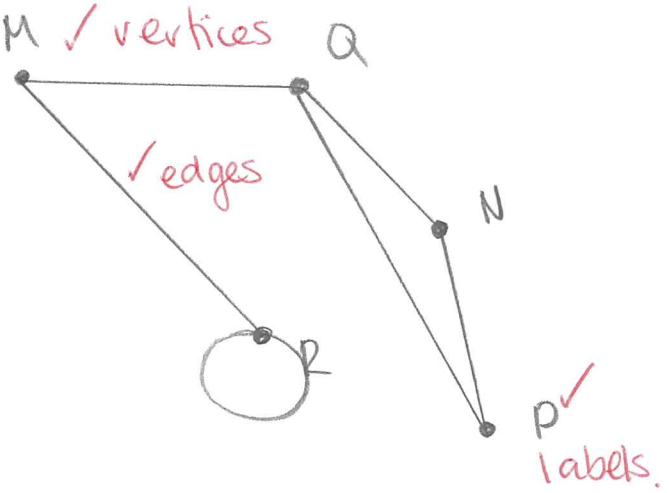


	P	Q	R	S
P	0	1	0	0
Q	1	0	2	0
R	0	0	0	1
S	2	1	1	0

✓ labels
✓ correct entries.

(b) Draw the graph for the following adjacency matrix. [3]

	M	N	P	Q	R
M	0	0	0	1	1
N	0	0	1	1	0
P	0	1	0	1	0
Q	1	1	1	0	0
R	1	0	0	0	1



(c) How many walks of length two are there from M to P? [1]

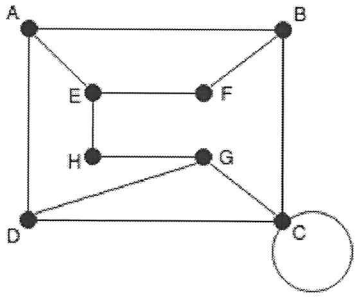
$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$ ✓ ans.

6

7

3. (9 marks)

The diagram below shows a graph.



(a) Determine, with reason, if this graph is a simple graph. [2]

No, contains a loop ✓ states loop

✓ ans no.

(b) Determine, with reason, if this graph is traversable. [2]

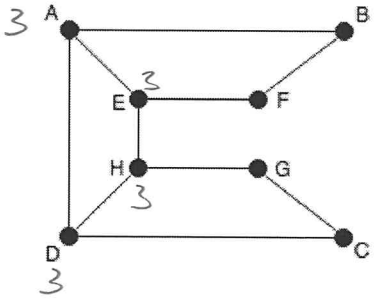
No, more than 2 odd vertices.

✓ ans. no. ✓ states odd vertices.

(c) Identify a Hamiltonian cycle for the above graph, giving your answer as a sequence of vertices. [1]

CBADGHEFC ✓

The graph has been altered to produce a subgraph.



(d) Determine, with reason, if the edge EH has become a bridge. [2]

No, if EH is removed it is still connected

✓ ans. no. ✓ states remains connected.

(e) Is the altered graph Eulerian, semi-Eulerian or neither? Justify you answer. [2]

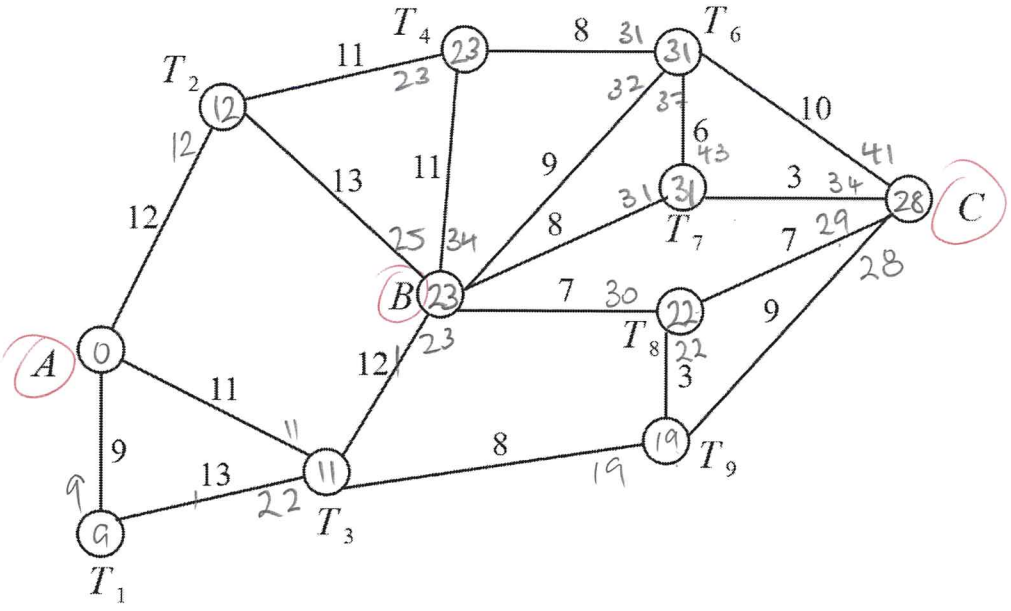
Neither, more than 2 odd vertices.

✓ states neither ✓ states odd vertices > 2.

7.

4. (5 marks)

The network below shows the connecting roads between three major towns, A, B and C. Jon lives in town A and his friend lives in town C. The weight represent distance, in kilometres between small towns and the three major towns A, B and C.



(a) If Jon would like to visit his friend in town C, what is the shortest distance he must travel? Determine Jon's path to town C. [3]

$A - T_3 - T_9 - C = 28$ km ✓ state distance.

✓ path

✓ working

(b) If Jon would like to pick up his friend Mark on his way in town B. What is the length of his short path now? Determine the path via town B. [2]

$A - T_3 \rightarrow B \rightarrow T_7 - C$ ✓ path.

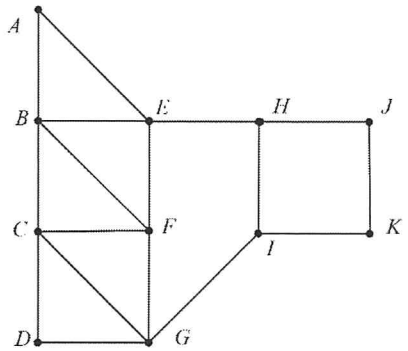
34 km ✓ states distance

5. (6 marks)

Toby, a mathematics student at Trinity College, was employed as a guide for a cultural tour of Perth. He decided to use graph theory to plan the walking tour.

Below is a network he constructed in which the:

- vertices represent points of interest to be visited, and
- edges represent the most direct route between adjacent vertices.



(a) Use Euler's formula to verify the network if connected. [1]

V = 11
F = 8
E = 17.

V + f - e = 2
11 + 8 - 17 = 2
✓ correct

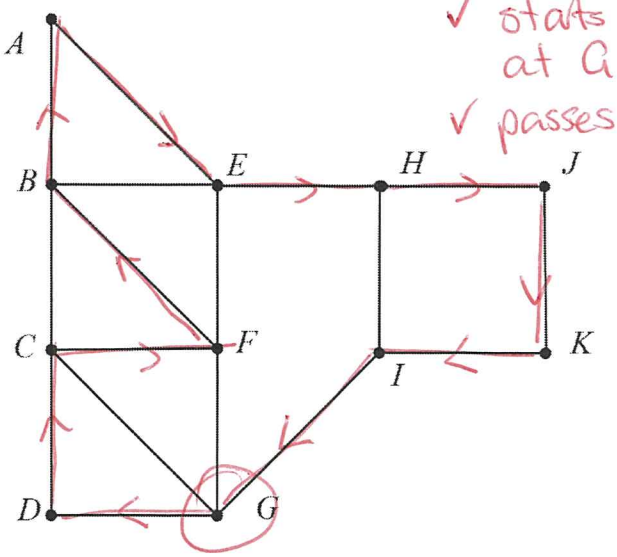
(b) Toby, planned to take the group on a closed walk. Explain the meaning of a closed walk. [1]

starts and finishes at the same vertex ✓

(c) He also stated that the walk would qualify as a Hamiltonian cycle. State the two properties that makes the walk a Hamiltonian cycle. [2]

- starts and finishes at same vertex ✓ starts + finishes.
- passes through each vertex once ✓ vertex once.

(d) Given that the walk started a Trinity College (G), mark the Hamiltonian cycle on the network below. [2]



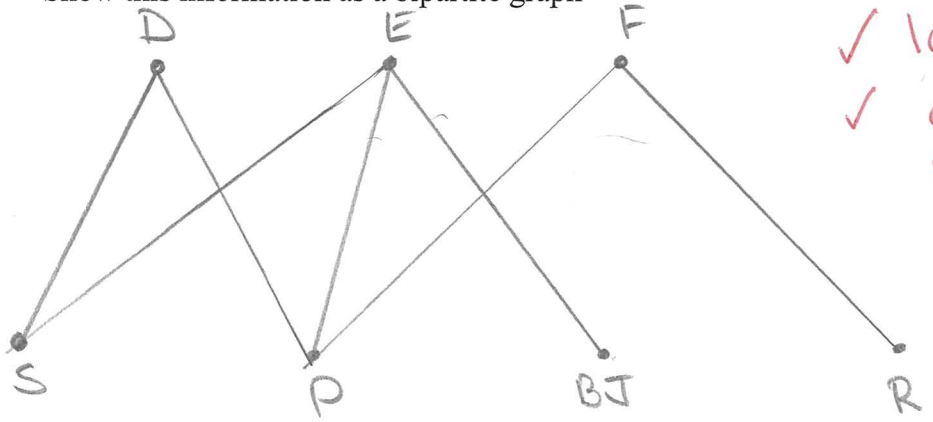
✓ starts + finishes at G.
✓ passes through each vertex once.

6. (4 marks)

(a) Three friends, David, Eddy and Frank, like to play different card games. [2]

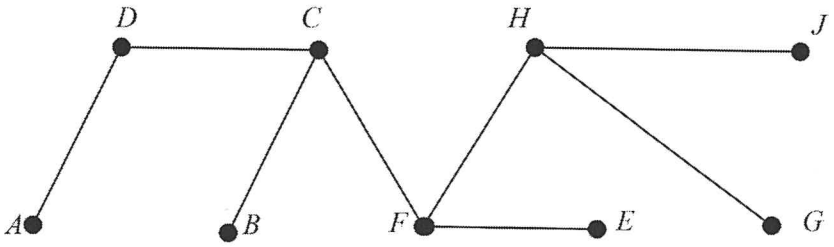
- David enjoys playing Snap and Poker
- Eddy enjoys playing Snap, Poker and Black Jack
- Frank enjoys playing Poker and Rummy

Show this information as a bipartite graph

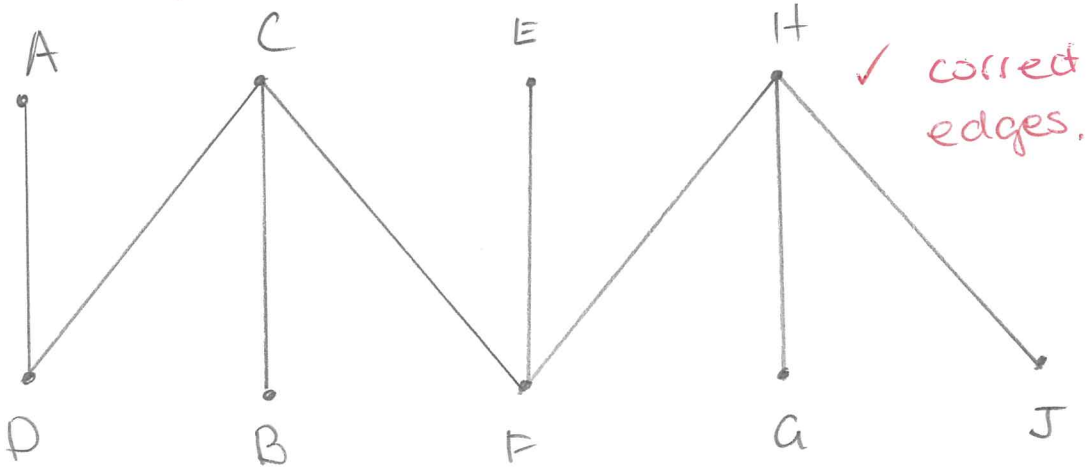


✓ labels
✓ correct number of edges.

(b) State whether the following graph is bipartite. If so, redraw the graph as a bipartite graph. [2]

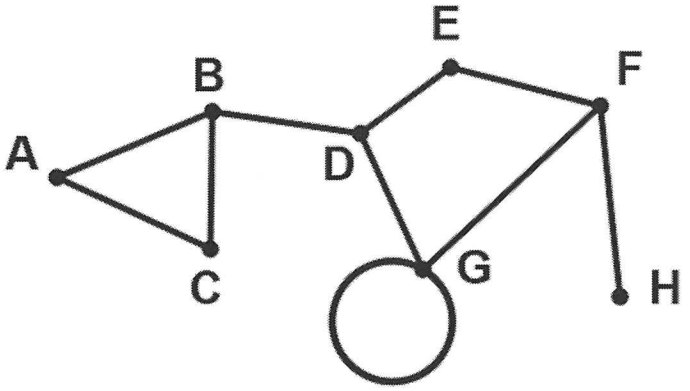


Yes ✓ states yes



✓ correct edges.

7. (6 marks)



Answer the following questions using the graph above.

(a) Verify that the graph obeys Euler's formula. [2]

$V = 8$

$f = 4$ ✓ correct values.

$e = 10$

$v + f - e = 2$

$8 + 4 - 10 = 2$.

✓ substitution

(b) List all the vertices that are of an odd degree. [2]

B, D, F, H.

✓ 3 correct.

✓ all correct

(c) State which edge(s) in the graph, if any are bridges. [2]

BD and FH.

✓ one correct

✓ both correct.